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A novel aerodynamic control for flutter instability of a long-span bridge by active flaps

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SUMMARY:
This paper proposes an active aerodynamic flutter control method for long-span bridge with controllable flaps. A theoretical framework for controlling scheme with a pair of rotatable flaps on the wind fairing of the deck is established, and a state space expression is derived and an optimal control algorithm is introduced. The movement of the flaps is determined by a feedback control algorithm. The active control system can improve the aerodynamic stability of long-span bridges through the aerodynamic forces generated by these flaps. Taking an ideal flat plate as an object, the flutter control of the flaps is verified through numerical simulation, and the controlling effectiveness is confirmed.

Keywords: active flap, feedback control, bridge flutter

1. INTRODUCTION
With the development of modern infrastructure long-span bridges have been proposed or under construction to cross sea or deep canyons. The difficulty of deep-water construction and requirement of shipping traffic require the modern bridges to cross these regions with a super-long single span. The continuous increment in span length and usage of advanced materials in these bridges greatly reduces their structural stiffness and damping, which results in a more wind sensitive structures. The flutter instability, which can cause bridge collapse, is an important consideration during the design process. Raggett et al. (1987) introduced an advanced active control method based on active flaps that originated in aeronautical engineering to improve the flutter stability of bridges. It has received intensive attention in the past ten years owing to the more flexibility of bridges and potentially stronger extreme wind events as a result of climate change (Ostenfeld and Larsen, 1997).

2. AERODYNAMIC FORCES MODELING
Theordorsen (1949) derived the unsteady aerodynamic expression of a wing-aileron-tab combination system with the potential flow assumption, and its theoretical solution can accurately simulate the aerodynamic lift and torque generated by the movement of the wing in

the air. This model can be transformed into an aerodynamic model with two active flaps attached to the wing, which can be used to describe flutter control models for most bridge decks installed with active flaps (Hansen, 2001), as shown in Figure 1.

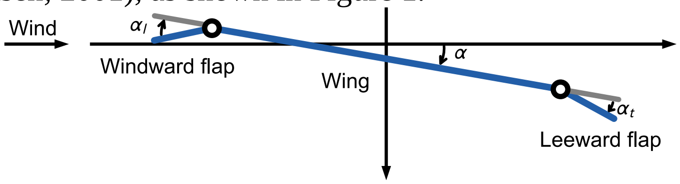

Figure 1. Simplified model with windward and leeward edge control flaps.

where α, α_l, and α_t are the torsion angles of the wing and windward flap and leeward flap, respectively. The self-excited force generated by the deck-flap system can be considered as the superposition of the respective aerodynamic forces of the deck, windward flap and leeward flaps. The expression in the form of flutter derivatives is shown as follows:

$$L = L_d + L_l + L_t = \frac{1}{2}\rho U^2 B \begin{bmatrix} KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B} \\ +KH_5^*\frac{B\dot{\alpha}_l}{U} + K^2H_6^*\alpha_l + KH_7^*\frac{B\dot{\alpha}_t}{U} + K^2H_8^*\alpha_t \end{bmatrix} \tag{1a}$$

$$M = M_d + M_l + M_t = \frac{1}{2}\rho U^2 B^2 \begin{bmatrix} KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B} \\ +KA_5^*\frac{B\dot{\alpha}_l}{U} + K^2A_6^*\alpha_l + KA_7^*\frac{B\dot{\alpha}_t}{U} + K^2A_8^*\alpha_t \end{bmatrix} \tag{1b}$$

where L_l, M_l, L_t and M_t represent the aerodynamic lift and torque on the windward and leeward flaps to the deck-flap system, respectively. ρ is the air density, U is the wind speed, B is the width of the deck-flap system, $K = \omega B/U$ is the reduced frequency, h and α are the vertical and torsional displacements of the deck, $\dot{h}$ and $\dot{\alpha}$ are the vertical and torsional speeds, respectively. α_l and α_t are the torsional displacements of the windward and leeward flaps, respectively; $\dot{\alpha}_l$ and $\dot{\alpha}_t$ are the torsional motion speeds of the windward and leeward flaps, respectively. H_{1-4}^*, A_{1-4}^*, H_{5-8}^* and A_{5-8}^* are the nondimensional flutter derivatives for the deck and flaps, respectively. However, the series of flutter derivatives at discrete frequencies are not sufficient in the control designs. To solve this problem, rational function approximation method is introduced, which is firstly presented by Roger (1977) to solve wing motion problems.

$$\mathbf{F}_{se}^{d} = \rho U^2 \left(\mathbf{b}^{\mathbf{T}}\mathbf{A_1}\mathbf{b}\mathbf{X} + \frac{B}{U}\mathbf{b}^{\mathbf{T}}\mathbf{A_2}\mathbf{b}\dot{\mathbf{X}} + \frac{B^2}{U^2}\mathbf{b}^{\mathbf{T}}\mathbf{A_3}\mathbf{b}\ddot{\mathbf{X}} + \textstyle\sum_{l=1}^{m}\mathbf{\Delta}_{l}\right) \tag{2a}$$

$$\mathbf{F}_{se}^{f} = \rho U^2 \left(\mathbf{b}^{\mathbf{T}}\mathbf{P_1}\mathbf{b}\mathbf{\Upsilon} + \frac{B}{U}\mathbf{b}^{\mathbf{T}}\mathbf{P_2}\mathbf{b}\dot{\mathbf{\Upsilon}} + \frac{B^2}{U^2}\mathbf{b}^{\mathbf{T}}\mathbf{P_3}\mathbf{b}\ddot{\mathbf{\Upsilon}} + \textstyle\sum_{l=1}^{m}\mathbf{\Theta}_{l}\right) \tag{2b}$$

$$\dot{\mathbf{\Delta}}_{l} = \frac{d_l U}{B}\mathbf{\Delta}_{l} + \dot{\mathbf{X}}, \dot{\mathbf{\Theta}}_{l} = \frac{g_l U}{B}\mathbf{\Theta}_{l} + \dot{\mathbf{\Upsilon}} \tag{3a, b}$$

where $\mathbf{\Delta}_{l}$ and $\mathbf{\Theta}_{l}$ are aerodynamic state variable; $\mathbf{X} = [h \quad \alpha]^T$ and $\mathbf{\Upsilon} = [\alpha_l \quad \alpha_t]^T$ assemble the displacements of deck and flaps, respectively. $\mathbf{A_1}$, $\mathbf{A_2}$ and $\mathbf{A_3}$ represent aerodynamic stiffness, damping and mass of deck, respectively; $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ represent aerodynamic stiffness, damping and mass of flaps, respectively; d_l and g_l represent decay rate of lag terms.

3. OPTIMUM CONTROL

Kinetic equation of this deck-flaps system can be expressed as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C_0}\dot{\mathbf{X}} + \mathbf{K_0}\mathbf{X} = \mathbf{F}_{se}^{d} + \mathbf{F}_{se}^{f} \tag{4}$$

where $\mathbf{M} = \begin{bmatrix} m & \\ & I \end{bmatrix}$, $\mathbf{C_0} = \begin{bmatrix} 2m\omega_h\xi_h & \\ & 2I\omega_\alpha\xi_\alpha \end{bmatrix}$ and $\mathbf{K_0} = \begin{bmatrix} \omega_h^2 m & \\ & \omega_\alpha^2 I \end{bmatrix}$. Substitute Eqs. (2) and (3) into Eq. (4). Kinetic equation of the deck can be re-expressed as

$$\bar{\mathbf{M}}\ddot{\mathbf{X}} + \bar{\mathbf{C}}\dot{\mathbf{X}} + \bar{\mathbf{K}}\mathbf{X} = \mathbf{\Gamma_1}\mathbf{\Upsilon} + \mathbf{\Gamma_2}\dot{\mathbf{\Upsilon}} + \mathbf{\Gamma_3}\ddot{\mathbf{\Upsilon}} + \rho U^2 \sum_{l=1}^{m} \mathbf{\Delta}_{\boldsymbol{l}} + \rho U^2 \sum_{l=1}^{m} \mathbf{\Theta}_{\boldsymbol{l}} \tag{5}$$

New matrix notations in Eq. (5) are given in Eqs. (6) and (7).

$$\bar{\mathbf{M}} = \mathbf{M} - \rho B^2 \mathbf{b}^{\mathbf{T}}\mathbf{A_3}\mathbf{b}, \bar{\mathbf{C}} = \mathbf{C_0} - \rho UB\mathbf{b}^{\mathbf{T}}\mathbf{A_2}\mathbf{b}, \bar{\mathbf{K}} = \mathbf{K_0} - \rho U^2\mathbf{b}^{\mathbf{T}}\mathbf{A_1}\mathbf{b} \tag{6}$$
$$\mathbf{\Gamma_1} = \rho U^2\mathbf{b}^{\mathbf{T}}\mathbf{P_1}\mathbf{b}, \mathbf{\Gamma_2} = \rho UB\mathbf{b}^{\mathbf{T}}\mathbf{P_2}\mathbf{b}, \mathbf{\Gamma_3} = \rho B^2\mathbf{b}^{\mathbf{T}}\mathbf{P_3}\mathbf{b} \tag{7}$$

The higher-order derivative terms in the equation should be eliminate when establishing a state-space representation, thus, it is necessary to retransform the kinetic equation which is given as Eq. (8).

$$\bar{\mathbf{M}}\dot{\mathbf{X}} + \bar{\mathbf{C}}\mathbf{X} + \bar{\mathbf{K}}\int \mathbf{X} = \mathbf{\Gamma_1}\int \mathbf{\Upsilon} + \mathbf{\Gamma_2}\mathbf{\Upsilon} + \mathbf{\Gamma_3}\dot{\mathbf{\Upsilon}} + \rho U^2 \sum_{l=1}^{m} \int \mathbf{\Delta}_{\boldsymbol{l}} + \rho U^2 \sum_{l=1}^{m} \int \mathbf{\Theta}_{\boldsymbol{l}} \tag{8}$$

The new state variables are selected as $\mathbf{Z} = \left[\boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2, \boldsymbol{\kappa}_{\Delta,1}, \cdots \boldsymbol{\kappa}_{\Delta,m}, \boldsymbol{\kappa}_{\Theta,1}, \cdots \boldsymbol{\kappa}_{\Theta,m}\right]^{T}$

$$\begin{cases} \boldsymbol{\kappa}_1 = \mathbf{X} - \bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}\mathbf{\Upsilon}, \ \ \boldsymbol{\kappa}_2 = \int\left(\mathbf{X} + \bar{\mathbf{K}}^{-1}\mathbf{\Gamma_1}\mathbf{\Upsilon}\right)\mathrm{dt} \\ \boldsymbol{\kappa}_{\Delta,1} = \int \mathbf{\Delta_1}\, dt, \cdots, \boldsymbol{\kappa}_{\Delta,m} = \int \mathbf{\Delta}_{\boldsymbol{m}}\, dt \\ \boldsymbol{\kappa}_{\Theta,1} = \int \mathbf{\Theta_1}\, dt, \cdots, \boldsymbol{\kappa}_{\Theta,m} = \int \mathbf{\Theta}_{\boldsymbol{m}}\, dt \end{cases} \tag{9}$$

Afterwards, the kinetic equations of the deck-flap system can be described as the state-space representation:

$$\begin{cases} \dot{\mathbf{Z}} = \boldsymbol{A_c}\mathbf{Z} + \boldsymbol{B_c}\mathbf{\Upsilon} \\ \mathbf{X} = \boldsymbol{C_c}\mathbf{Z} + \boldsymbol{D_c}\mathbf{\Upsilon} \end{cases} \tag{10}$$

$$\boldsymbol{A_c} = \begin{bmatrix} -\bar{\mathbf{M}}^{-1}\bar{\mathbf{C}} & -\bar{\mathbf{M}}^{-1}\bar{\mathbf{K}} & \bar{\mathbf{M}}^{-1}\rho U^2 & \cdots & \bar{\mathbf{M}}^{-1}\rho U^2 & \bar{\mathbf{M}}^{-1}\rho U^2 & \cdots & \bar{\mathbf{M}}^{-1}\rho U^2 \\ \mathbf{I} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \mathbf{b}^{\mathbf{T}}\mathbf{A_4}\mathbf{b} & 0 & -\frac{d_1 U}{B}\mathbf{I} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{b}^{\mathbf{T}}\mathbf{A}_{\boldsymbol{m}}\mathbf{b} & 0 & 0 & \cdots & -\frac{d_m U}{B}\mathbf{I} & 0 & \cdots & 0 \\ \mathbf{b}^{\mathbf{T}}\mathbf{P_4}\mathbf{b} & 0 & 0 & \cdots & 0 & -\frac{g_1 U}{B}\mathbf{I} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{b}^{\mathbf{T}}\mathbf{P_m}\mathbf{b} & 0 & 0 & \cdots & 0 & 0 & \cdots & -\frac{g_m U}{B}\mathbf{I} \end{bmatrix} \tag{11a}$$

$$\boldsymbol{B_c} = \left[\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_2} - \bar{\mathbf{M}}^{-1}\bar{\mathbf{C}}\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}, \ \bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3} - \bar{\mathbf{K}}^{-1}\mathbf{\Gamma_1}, \ \mathbf{b}^{\mathbf{T}}\mathbf{A_4}\mathbf{b}\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}, \cdots, \mathbf{b}^{\mathbf{T}}\mathbf{A}_{\boldsymbol{m}}\mathbf{b}\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}, \mathbf{b}^{\mathbf{T}}\mathbf{P_4}\mathbf{b}\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}, \cdots, \mathbf{b}^{\mathbf{T}}\mathbf{P_m}\mathbf{b}\bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3}\ \right]^{T} \tag{11b}$$
$$\boldsymbol{C_c} = [\mathbf{I} \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad \cdots \quad 0] \tag{11c}$$
$$\boldsymbol{D_c} = \bar{\mathbf{M}}^{-1}\mathbf{\Gamma_3} \tag{11d}$$

The motion of the flaps is determined by $\mathbf{\Upsilon} = -\mathbf{KZ}$, and $\mathbf{K}$ is the feedback gain matrix. The deck-flap system can be regarded as a linear time-invariant system and transformed into an infinite-time regulator problem. Thus, the objective optimization function is

$$J = \frac{1}{2}\int_0^{+\infty}\left(\mathbf{Z}^{\mathrm{T}}\mathbf{QZ} + \mathbf{\Upsilon}^{T}\mathbf{R\Upsilon}\right) dt \tag{12}$$

where, $\mathbf{Q}$ and $\mathbf{R}$ represent the constant weight matrix, and finally can be solved by a Riccati equation which is given as

$$\mathbf{Y}\boldsymbol{A_c} + \boldsymbol{A_c^T}\mathbf{Y} - \mathbf{Y}\boldsymbol{B_c}\mathbf{R}^{-1}\boldsymbol{B_c^T}\mathbf{Y} + \boldsymbol{Q} = 0, \mathbf{K} = \mathbf{R}^{-1}\boldsymbol{B_c^T}\mathbf{Y} \tag{13a, b}$$

where $\mathbf{Y}$ is an undetermined constant matrix.

4. NUMERICAL CASE

According to the expression of state space and the optimal control method, the feedback control parameters of the deck-flap system can be obtained. Taking the ideal plate with flaps as the object to simulate the movement before and after the control, the torsional displacement is shown in Figure 2.

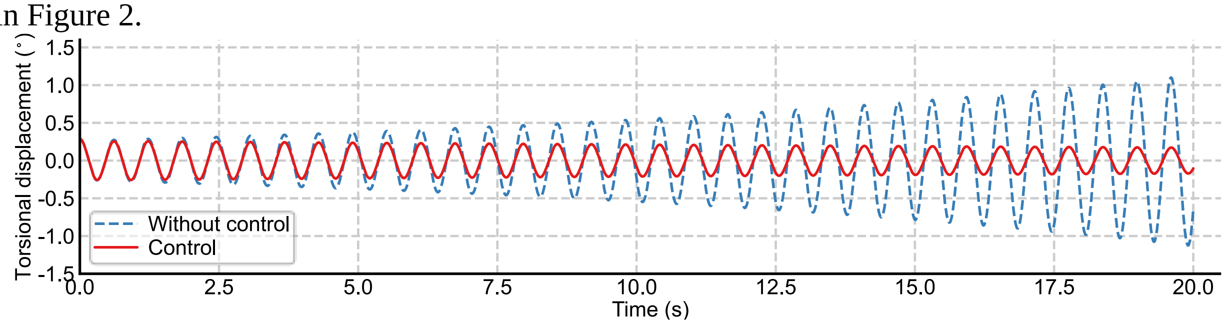

Figure 2. Torsional displacement of deck-flap system with and without control (U=critical wind speed of the flutter)

5. CONCLUSIONS

The aerodynamic state space expression of the deck-flap system is derived, and the flutter control simulation is carried out with an ideal flat plate as the example. The method of obtaining aerodynamic parameters of the actual bridge section and the control effect of the active flaps will be finished in further work and presented in full paper.

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REFERENCES

Raggett J D. Stabilizing winglet pair for slender bridge decks[C]//Bridges and Transmission Line Structures. ASCE, 1987: 292-302.

Ostenfeld K H, Larsen A. Elements of active flutter control of bridges[J]. Proc. of New Technologies in Structural Engineering, LNEC, Lisbon, 1997: 683-694.

Theodorsen T. General theory of aerodynamic instability and the mechanism of flutter[R]. 1949.

Hansen H I, Thoft‐Christensen P. Active flap control of long suspension bridges[J]. Journal of Structural Control, 2001, 8(1): 33-82.

Roger K L. Airplane math modeling methods for active control design[J]. 1977.